Cormack's implicit scheme made in the original paper, although algebraically correct, does not seem to be the most suitable. Due to its resemblance to the explicit scheme, the following parameters are proposed: $\xi = \frac{1}{2}$, θ is related to the parameter λ introduced in the MacCormack scheme [Ref. 3, Eq. (5)] by $\theta = \lambda / |a|$. With MacCormack's choice,

$$\lambda = \max\left\{ |a| - \frac{1}{2\sigma},\ 0 \right\}$$

This yields

$$\theta = \max\left\{ 1 - \frac{1}{2\sigma |a|},\ 0 \right\}$$

This choice of parameters appears more convenient in the sense that when the implicit option is not used in MacCormack's scheme ($\lambda = 0$), the parameter θ is zero.

Finally, since according to the stability analysis in Ref. 1, the optimum choice for the dissipation of numerical modes is, in this case,

$$\theta = \frac{\sqrt{2}}{2} - \frac{1}{2\sigma |a|}$$

the optimum choice for a scheme taking advantage of the explicit option whenever possible from stability considerations would be

$$\theta = \max\left\{ \frac{\sqrt{2}}{2} - \frac{1}{2\sigma |a|},\ 0 \right\}$$

References

[1]Casier, F., Deconinck, H., and Hirsch, Ch., "A Class of Bidiagonal Schemes for Solving the Euler Equations," *AIAA Journal*, Vol. 22, Nov. 1984, pp. 1556-1563.

[2]MacCormack, R. W., "The Effect of Viscosity in Hypervelocity Impact Cratering," AIAA Paper 69-354, April 1969.

[3]MacCormack, R. W., "A Numerical Method for Solving the Equations of Compressible Viscous Flow," *AIAA Journal*, Vol. 20, Sept. 1982, pp. 1275-1281.

Reply by Authors to G. Degrez

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THE authors[1] generally agree with this Comment. Indeed, the central bidiagonal scheme (CBS) contains two parameters (θ and ξ), whereas the McCormack implicit scheme contains only one parameter (λ). This allows the free choice of one of the CBS parameters (e.g., ξ) for the identification of both schemes.

The choice of $\xi = \frac{1}{2}$ made by Degrez is indeed preferable if one wishes to compare or to switch to the explicit McCormack scheme obtained for $\xi = \frac{1}{2}$, $\theta = 0$. In particular, this choice allows the traditional interpretation of the space derivatives in the McCormack implicit scheme as one-sided downwind in the U-sweep predictor step [discretized at $i + \frac{1}{2} - \xi = i$ using cell $(i, i+1)$] and one-sided upwind in the L-sweep corrector step [discretized at $i + \frac{1}{2} + \xi = i + 1$ using cell $(i, i+1)$]. As a result, each step is first-order accurate in space. Note that the McCormack scheme as described in Ref. 2 of the Comment is a U-L scheme with sweeps from right to left in the predictor and left to right in the corrector step, explaining the reversed sign of ξ compared to the L-U scheme described in our paper.[1]

In the context of the paper, however, the choice $\xi = 0$ leading to $\theta = 1$ has some attractive aspects: The type of calculations made by the authors uses a constant local CFL number in each meshpoint, equal to the value specified in the data file. Hence, the explicit stability condition is never satisfied in any point of the mesh and application of the McCormack implicit scheme would never switch to the explicit scheme in this case. Thus, the choice of $\theta = 1$ is independent of the CFL number, as opposed to the choice made in the Comment.

Further, the choice $\xi = 0$, $\theta = 1$ shows the close resemblance with the optimal fully implicit scheme used in the numerical tests in the paper and determined by the choice $\xi = 0$, $\theta = \sqrt{2}/2 = \pm 0.7$.

Finally, the choice $\xi = 0$, $\theta = 1$ shows that the McCormack implicit scheme can be interpreted as resulting from a central second-order discretization in the point $i + \frac{1}{2}$ in both predictor and corrector steps (if no use is made of the explicit option). This central "box" interpretation of the McCormack implicit scheme strongly differs from the usual interpretation and can have important consequences, e.g., in the presence of a source term as in the quasi-one-dimensional Euler equations. The source term for the scheme $\xi = \frac{1}{2}$ would be discretized at i in the predictor step and at $i + 1$ in the corrector step, precisely as in the traditional explicit McCormack scheme. For the scheme $\xi = 0$, however, the source term would be discretized at $i + \frac{1}{2}$ in both the predictor and corrector step,[1] which is obtained by taking the average over the values at i and $i + 1$. On the other hand, with $\xi = 0$, each step is second-order accurate in space at $i + \frac{1}{2}$.

Again, the identification proposed by Degrez is more in line with the traditional McCormack approach.

References

[1]Casier, F., Deconinck, H., and Hirsch, Ch., "A Class of Bidiagonal Schemes for Solving the Euler Equations," *AIAA Journal*, Vol. 22, Nov. 1984, pp. 1556-1563.

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Comment on "Application of the Generalized Inverse in Structural System Identification"

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THE use of the Moore-Penrose generalized inverse underlies much of the current work in structural dynamics. Whereas Chen and Fuh[1] and Berman[2,3] use this inverse explicitly (and correctly) in model adjustment using identified modes, it is often also used implicitly in the identification algorithms that provide the data for model adjustment. In this application it is not always clear that the analyst is aware of the limitations of the method. The essential difference is that Chen and Fuh[1] are able to assume in their analysis that "the measured modal matrix Φ ($n \times m$) is rectangular with full column rank m." In the identification stage, however, the rank of the modal matrix corresponds to the number of identifiable modes in the test data. Under many test conditions the decision as to the number of identifiable modes is not clear-cut and depends on the subjective judgment of the analyst.

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Mergeay,[4] for example, demonstrates a considerable degree of uncertainty in the exact number of modes identified in practical tests. Incorrect estimation of the rank of the measured modal matrix will lead either to deletion of genuine structural modes if rank is underestimated or to the identification algorithm inventing its own modes (ghost modes). Either of these errors might lead to serious discrepancies in the adjustment process used by Chen and Fuh. The key to this problem is the use of the weighting matrix $\underset{\sim}{W}$, which Chen and Fuh suggest may be "any symmetric, nonsingular weighting matrix," The weightings should, in practice, reflect the confidence of the analyst in each identified mode.[5]

In conclusion it must be observed that, in using the generalized inverse it is advisable to use proven algorithms. The documentation for the NAg[6] library suggests:

"...only the singular value decomposition gives a reliable indication of rank deficiency....Sound decisions can only be made by somebody who appreciates the underlying physical problem."

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Reply by Authors to J.A. Brandon

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WE wish to thank Dr. Brandon for his interest and comment concerning our Note.[1] Our answers to the posed problems follow.

The first question Dr. Brandon raised concerned the rank of measured modal matrix $\Phi(n\times m)$. In our Note, the full rank assumption was made because we recognized the necessity of engineering judgment in adopting the correct modes before applying model improvement procedures. The rank problem can be viewed from several facets. As far as rank determination is concerned, well-documented numerical algorithms (Gauss elimination, Householder transformation, singular-valued decomposition, etc.) are available for this purpose. The accuracy of rank depends entirely on the algorithm (and specified tolerances) used, provided the modal matrix is not contaminated with errors. Of course, the noise-free environment does not exist in vibration test and modal extraction stages. The measurement uncertainties are expected to be greater than computational errors introduced by the rank determination algorithm. In other words, determination of the rank of a measured modal matrix requires interactive checks and subjective judgment which no "reliable" algorithm can replace.

The basis of our identification method is a well-formulated analytical model and a modal matrix in which we have engineering confidence. Once large model changes result from the improvement procedure, then either the analytical model or Φ, or both, are bad. In order not to digress from our subject, consider the case where the analytical model is a good representation of the structure. An expository account of the relationship between rank and model changes is useful for the understanding of our approach. Let ϕ_i and ϕ_j be two almost, but not exactly, identical vectors of Φ whose differences are large enough to avoid a rank deficiency problem. Normalization of ϕ_i and ϕ_j will lead to $\phi_i^T M_a \phi_i = 1 = \phi_j^T M_a \phi_j$, where M_a is the (unimproved) analytical mass matrix. Obviously, we also have $\phi_i^T M_a \phi_j$ and $\phi_j^T M_a \phi_i$ close to 1. That is, $\Phi^T M_a \Phi$ will make two off-diagonal elements (i,j) and (j,i) approximately equal to 1. However, since the imposed orthogonality condition, $\Phi^T M \Phi = I$ (M, the improved mass matrix), forces all off-diagonal elements to be zero, this constraint would be too rigid and consequently drive the mass changes high. Similar arguments also apply to stiffness changes where the dynamic equation is the imposed constraint. Therefore, we conclude that a "ghost" mode corresponds to large model changes. Physical insight can be gained by using several mode combinations and comparing the associated model changes as made in Ref. 2.

Once the "independent" modes have been selected, they are treated as exact, which is a basic assumption made in the Note. That is, minimum changes on the model should be found to force the improved model exactly satisfying the orthogonality condition and dynamic equation. In mathematical terms, the resultant system is consistent and well defined because the number of degrees of freedom n is much larger than the number of modes of interest m. Following this reasoning, the choice of the weighting matrix does not "reflect the confidence of the analyst in each identified mode" as Dr. Brandon believes and, thus, has nothing to do with rank deficiency. The method in the Note is quite different from the ordinary least-square approach where the confidence level in each mode is used to adjust the error distribution and may have significant effects on the improved model. The weighting matrix given in the Note, however, is (in a sense) used to introduce the confidence level in each element that is subjected to change during the minimum-norm identification procedure. Dr. Brandon may have overlooked the difference between these two approaches, i.e., minimum-norm vs least-square.

In the last paragraph of his Comment, Dr. Brandon seemed to suggest that the singular valued decomposition (SVD) is the only reliable algorithm in computing the Moore-Penrose inverse A^+ of an $n\times m$ matrix of rank r. SVD is well known for its numerical stability. Unfortunately, since it invokes the eigen solutions of an $n\times n$ matrix, AA^T, and an $m\times m$ matrix, A^TA, SVD is very inefficient for analytical model improvement where n is of hundreds or thousands. A much more efficient and equally powerful algorithm is the Householder transformation, used to decompose A into the form QU, where Q is an $n\times r$ orthogonal matrix and U an upper triangular.[3] Then, the Moore-Penrose inverse A^+ reads $U^T(UU^T)^{-1}Q^T$. This algorithm gains its efficiency and reliability from the following: 1) complete information of A^+ is obtained from decomposition of A (no products like AA^T or A^TA in SVD are required); 2) the only regular inverse performed is UU^T, a matrix of order r; and 3) the Householder transformation is numerically stable and the factored matrices can be efficiently stored.

Submitted April 4, 1985.

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